Physics 2: Special Relativity *

Igor Bray <I.Bray@curtin.edu.au>
Head, Physics and Astronomy
Curtin University
Perth, Western Australia

Last update: November 17, 2025
https://atom.curtin.edu.au/SR.pdf

*https://atom.curtin.edu.au/ModernPhysics

Contents

1 Preliminaries

- Einstein: "Education is what is left after everything learned has been forgotten"
  - Leads to transferable skills for success in life
  - Fights 2nd Law of Thermodynamics: Entropy (S) increases; Information ($I = 1/S$) diminishes (for isolated systems).
  - Study Physics because it
    - is hard; requires courage, builds confidence
    - is evidence-based: "You are not entitled to any opinion, only what you can argue for!"
    - builds knowledge depth; know if don't know!
    - leads to effective action
    - welcomes failure, builds resilience
    - requires balance of clarity and brevity
    - is motivated by autonomy, mastery, purpose
- "Lecturing is the process by which the notes of the lecturer get transferred to the notes of the student bypassing the brains of either!"
  - Harvard's Eric Mazur's interactive teaching.
  - Please familiarise yourself with the lecture notes before class!

- Importance of clear and succinct notation:

  An object moves with velocity $v(t)$ from $x(t_i)$ to $x(t_f)$ with *constant* acceleration a. By definition, $a = [v(t_f) - v(t_i)]/[t_f - t_i]$. Alternatively, $a \equiv \frac{dv}{dt}$

$$\begin{aligned}\int_{t_i}^{t_f} \frac{dv}{dt} dt &= a \int_{t_i}^{t_f} dt \\ v(t)\Big]_{t_i}^{t_f} &= a\,t\Big]_{t_i}^{t_f} \\ v(t_f) - v(t_i) &= a[t_f - t_i]. \end{aligned} \tag{1.1}$$

  Either way, we have

$$v(t_f) = v(t_i) + a[t_f - t_i]. \tag{1.2}$$

  However, High Schools (used to!) write $v = u + at$; four symbols with three mistakes!

  1. Show functional dependence, e.g. $v(t)$ to distinguish from constants such as a,
  2. Do not invent new symbols for same concepts, e.g. use $v(t_i) \equiv v_i$ rather than u,
  3. Time difference $t_f - t_i \equiv \Delta t$ is not time t.

  Exercise: for $v(t) \equiv \frac{dx}{dt}(t)$, integrate (1.2) to yield

$$\Delta x \equiv x(t_f) - x(t_i) = v(t_i)\Delta t + \frac{1}{2} a \Delta t^2. \tag{1.3}$$

2 Introduction

- Theory of Relativity was published by Einstein in 1905 and deals with motion in the absence of gravity. It became known as Special Relativity after General Relativity, which includes gravity, was introduced in 1915.

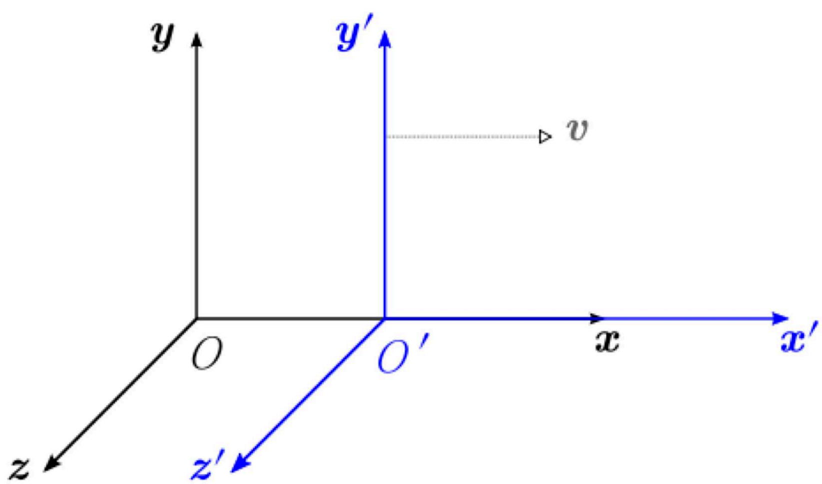
- Special Relativity deals with space and time:

  - What is space? Three dimensions x, y, z. Generally, no preferred direction. Increasingly expanding since the Big Bang. Is space real?
  - What is time? Einstein: "That which is measured by clocks!". Can write an object's position (x, y, z) in parametric form $x(t), y(t), z(t)$.
    - Is time real?
    - Does time exist if nothing changes?
    - What was there before The Big Bang?
    - Is there something special about the present moment?
    - What is the difference between the past and the future?
    - Time is inextricably connected to the speed of light; $\Delta t = \Delta x / c$.

3 Galilean Relativity

- Galileo: The laws of mechanics must be the same in all inertial (non-accelerating) frames of reference.
  - Inertial frames $S(t, x, y, z)$ and $S'(t', x', y', z')$ have relative constant velocity $v \equiv \frac{dx}{dt}$ with x-axes aligned and $O' = O$ at $t = 0$. Then

$$\begin{aligned} t' &= t \\ x'(t') &= x(t) - vt \\ y'(t') &= y(t) \\ z'(t') &= z(t) \end{aligned} \tag{3.1}$$



  - Note: v is a vector ($\boldsymbol{v}$), use $\pm$ for direction.
  - What is the velocity of O relative to O'?
  - If O' is approaching O, is v positive, negative, or can't tell?

- Interchange frames: $S \leftrightarrow S'$ with $v \leftrightarrow -v$.
- An object (ball) in S' has $u_{x'} \equiv \frac{dx'}{dt'} = \frac{dx}{dt} - v$, i.e.

$$u_{x'} = u_x - v, \text{ or } u_x = u_{x'} + v \tag{3.2}$$

  are the Galilean velocity addition formulas.
- For general $\boldsymbol{u}(t)$, $u_{y'}(t') = u_y(t)$ and $u_{z'}(t') = u_z(t)$.
- Under Galilean transformations:
  - Lengths are invariant. Let $x_1 = x_0$, $x_2 = x_0 + l$ then

$$\begin{aligned} l' &= x'_2 - x'_1 \\ &= (x_2 - vt) - (x_1 - vt) \\ &= (x_0 + l - vt) - (x_0 - vt) \\ &= l. \end{aligned}$$

  - Forces are invariant since typically depend on the distance between objects $(x_2 - x_1)$.
  - Laws of physics are covariant. Conservation of momentum in S also holds in S'. Two masses moving along x stick together after collision:

$$\begin{aligned} m_1 u_1 + m_2 u_2 &= (m_1 + m_2) u_3, \\ m_1(u'_1 + v) + m_2(u'_2 + v) &= (m_1 + m_2)(u'_3 + v), \\ m_1 u'_1 + m_2 u'_2 &= (m_1 + m_2) u'_3. \quad (3.3) \end{aligned}$$

4 Special Relativity

4.1 Postulates

1. All laws of physics have the same form in all inertial (relative velocity v is constant) frames.[1]

2. The speed of light, $c \approx 3 \times 10^8$ m/s in a vacuum, is constant.[2]

- General Relativity adds just one more postulate: In a local neighbourhood gravitational and inertial forces are equivalent.

- The ultimate goal is to reformulate the Laws of Physics in a covariant form: i.e. be invariant under transformation between inertial frames of arbitrary relative velocity $v < c$.

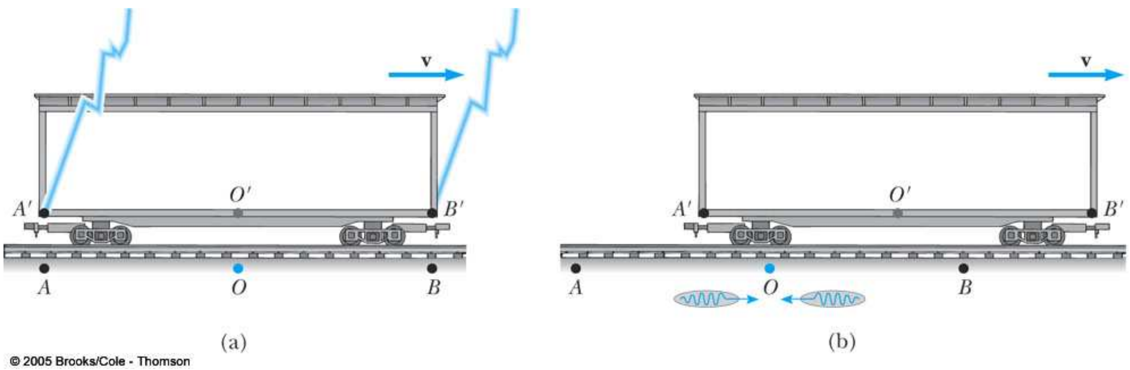
[1] Laws of Physics arise from symmetry principles (see 1915 Emmy Noether's theorem), and so do the Laws of societies!

[2] $c \approx 3$ Å/as, where angstrom Å $\equiv 10^{-10}$ m, and attosecond as $\equiv 10^{-18}$ s.

4.2 Simultaneity

Let us consider the consequences of the 2nd postulate.

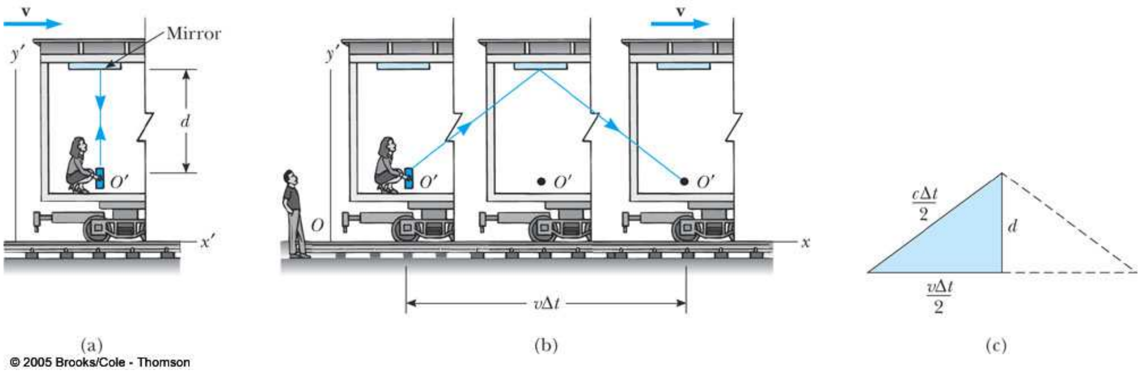
- Simultaneous events in one frame are not simultaneous in a frame moving relative to the first, see figure below.



  - O sees events A and B as simultaneous, see (b) above, but O' sees event B' before A'.
  - Hence, time intervals are different in frames moving relative to each other.
  - This is due to the *finite* speed of light.
  - Our intuition is based on essentially an infinite speed of light.

4.3 Time dilation

Consider the figures (a), (b) and (c) below.



(a) Within the train frame S', light travels a distance (up plus down) $c\Delta t' = 2d$, or $d = c\Delta t'/2$.

(b) In the outside frame S, light travels (up plus down) distance $c\Delta t$ and the horizontal distance $v\Delta t$.

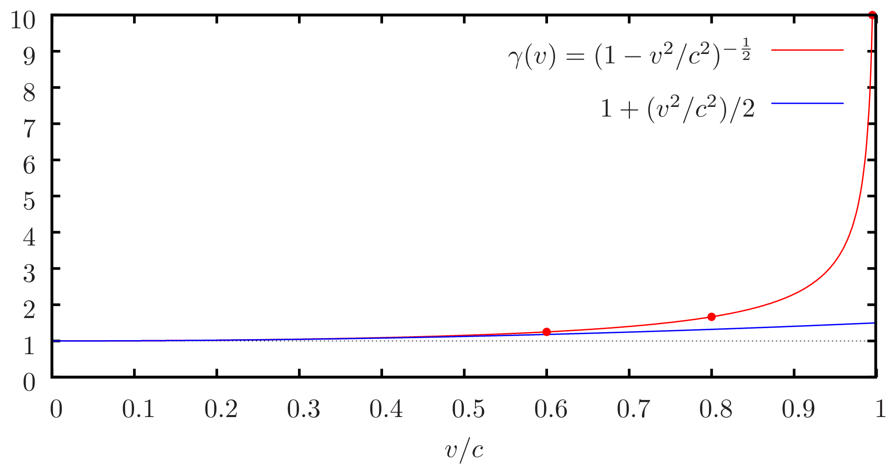
(c) Hence $(c\Delta t/2)^2 = d^2 + (v\Delta t/2)^2$.

- Substituting $(c\Delta t'/2)^2$ for d^2, and solving for Δt, leads to $\Delta t = \Delta t'/\sqrt{1-\frac{v^2}{c^2}}$.
- The time in a frame at rest with the clock is known as the *proper time* $\Delta t_{\mathrm{p}} = \Delta t' \leq \Delta t$ (dilates for moving frames), and so

$$\Delta t = \gamma \Delta t_{\mathrm{p}}, \text{ where } \gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \geq 1. \quad (4.1)$$

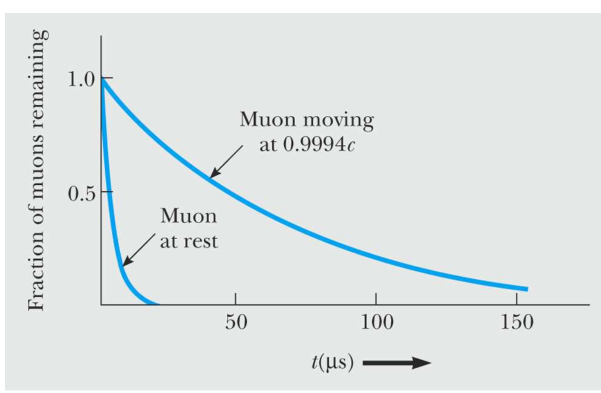
- γ and its (two-term) Taylor Series expansion using

$$(1+x)^n \approx 1 + nx \text{ for } |x| \ll 1, n \in \mathbb{R} \qquad (4.2)$$



  - for $\beta = v/c = 3/5 \iff \gamma(v) = 5/4$
  - for $\beta = v/c = 4/5 \iff \gamma(v) = 5/3$
  - for $\beta = v/c = 0.995 \iff \gamma(v) \approx 10.0$

- Time dilation of moving particles is consistent with experimental observation. Not proof of correctness.



  - Sometimes, theory can falsify experiment!

4.4 Length contraction

- Define the proper length $L_{\rm p}$ as the length measured in the rest frame.

- If O' travels $L_{\rm p} = v\Delta t$ relative to O, see fig. pg. 6, then O travels $-L' = -v\Delta t_{\rm p}$ relative to O'.

- Have $v = L'/\Delta t_{\rm p} = L_{\rm p}/\Delta t$, and using Eq. (4.1)

$$L' = L_{\rm p}/\gamma. \tag{4.3}$$

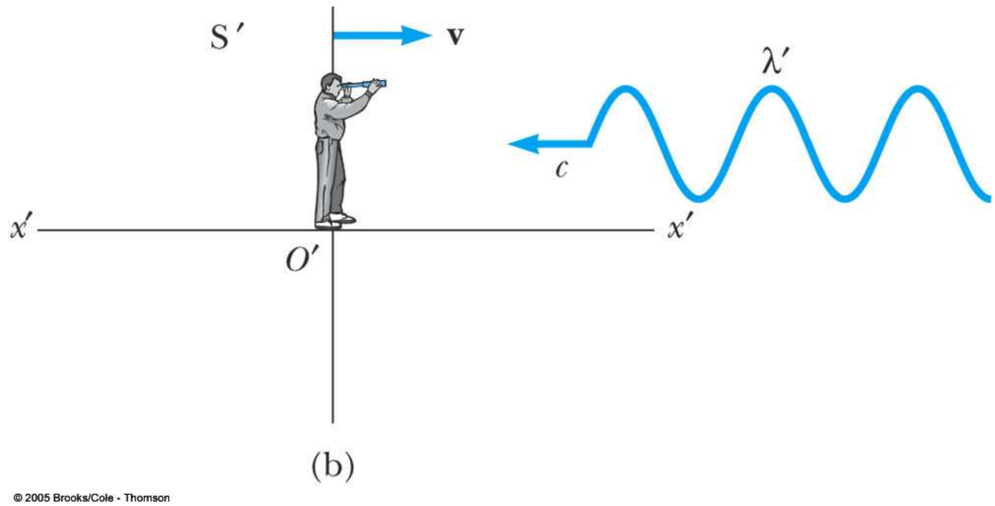
- Note $L' \leq L_{\rm p}$, i.e. lengths contract along the direction of motion.

- Muons of (proper) half-life $\Delta t_{\rm p} \sim 10^{-6}$ s ($c\Delta t_{\rm p} =$ 300 m) created at 5 km above the Earth, traveling at 0.995 c ($\gamma \approx 10$), reach the surface in large numbers:

  - Muon frame: small half-life, but travels short distance (5,000 m/10=500 m).
  - Earth frame: Ten times longer half-life, so can travel longer (3,000 m) distance.
  - More details and YouTube video.

4.5 Relativistic Doppler effect

- Stationary observer O (with telescope) receiving light of emitted wavelength λ' from receding O' with velocity v



- The wavelength of light λ is the distance between successive wavefronts and is related to the frequency $f \equiv 1/T$ via $c = f\lambda$, i.e. $\lambda = cT$.

- From perspective of O', moving away from O at speed v then $T' = \lambda'/(c-v)$, and

$$\begin{aligned} \lambda' &= (c-v)T' \\ &= (c-v)\gamma T \\ &= \frac{(c-v)}{\sqrt{1-\frac{v^2}{c^2}}}\frac{\lambda}{c} \end{aligned}$$

$$= \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\lambda, \tag{4.4}$$

or for frequencies $f' = f_\mathrm{s}$ (source) have

$$\frac{f_\mathrm{s}}{f} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}. \tag{4.5}$$

- Redshift $z = (f_\mathrm{s} - f)/f$

$$z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} - 1 \approx \frac{v}{c} \text{ for } v \ll c, \tag{4.6}$$

  where we used the Taylor Series expansion (4.2).

- Hence have increased f (blue-shift, $z < 0$) for approaching ($v < 0$) light sources, and decreasing f (redshift, $z > 0$) for receding sources of light.

- More info.

End of first week's prereading...

5 Lorentz Transformations

- Lorentz transformations generalise Galilean transformations (3.1), keeping $y' = y$, $z' = z$, to build in length contraction and time dilation

$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \quad ct' = \gamma\left(ct - \frac{v}{c}x\right) \qquad (5.1)$$

$$x' = \gamma(x - vt), \qquad x' = \gamma\left(x - \frac{v}{c}ct\right). \qquad (5.2)$$

- Write in matrix form as

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\frac{v}{c} & 0 & 0 \\ -\gamma\frac{v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}. \qquad (5.3)$$

- Inverse Lorentz transformations are obtained by interchanging the primes and changing the sign of v. Check for (ct, x) using matrix inversion.

- In general, define four-vector (tensor[3]) $x_\mu \equiv (ct, x, y, z)$ with transpose x^μ, $\mu = 0, 1, 2, 3$, and write (5.3) as

$$x^{\mu'} = L^{\mu'}{}_{\mu} x^\mu \equiv \sum_{\mu=0}^{3} L^{\mu'}{}_{\mu} x^\mu, \qquad (5.4)$$

where $L^{\mu'}{}_{\mu}$ are the matrix elements in Eq. (5.3).

[3] A tensor is an object with specified coordinate transformation properties.

5.1 Minkowski diagrams

- Minkowski Diagrams incorporate the constancy of the speed of light and the Lorentz transformations

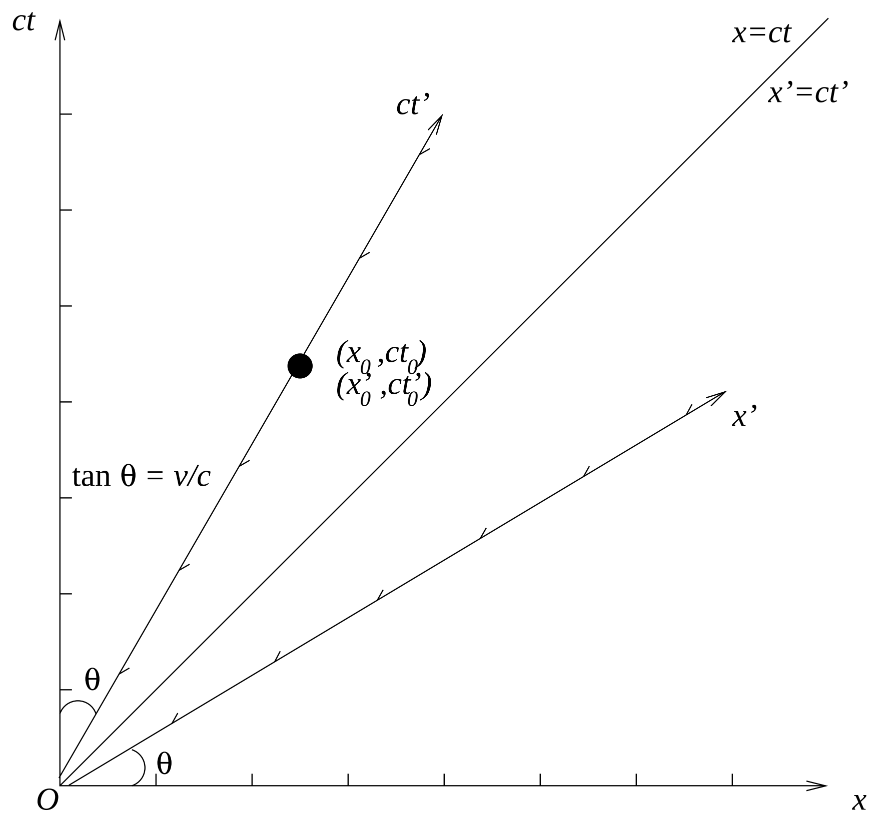


Figure 1: Minkowski diagram with $v/c = 1/\sqrt{3}$, and hence $\theta = 30^\circ$ and $\gamma = \sqrt{3/2}$. Axes have units of length, e.g. light-years.

- Inverse of (5.1): axis $ct' = ct/\gamma$ since $x' = 0$.
- Inverse of (5.2): axis $x' = x/\gamma$ since $t' = 0$.
- The ct' axis has $x' = 0 = \gamma(x_0 - vt_0)$. Hence, $\tan\theta = x_0/(ct_0) = vt_0/(ct_0) = v/c$.

- Time dilation:

  - Consider two events at $x_1 = x_2$ at times t_1 and t_2. Using Eq. (5.1) $t'_1 = \gamma(t_1 - \frac{vx_1}{c^2})$ and $t'_2 = \gamma(t_2 - \frac{vx_1}{c^2})$. Hence,

$$\Delta t' = t'_2 - t'_1 = \gamma(t_2 - t_1) = \gamma \Delta t_{\mathrm{p}}. \quad (5.5)$$

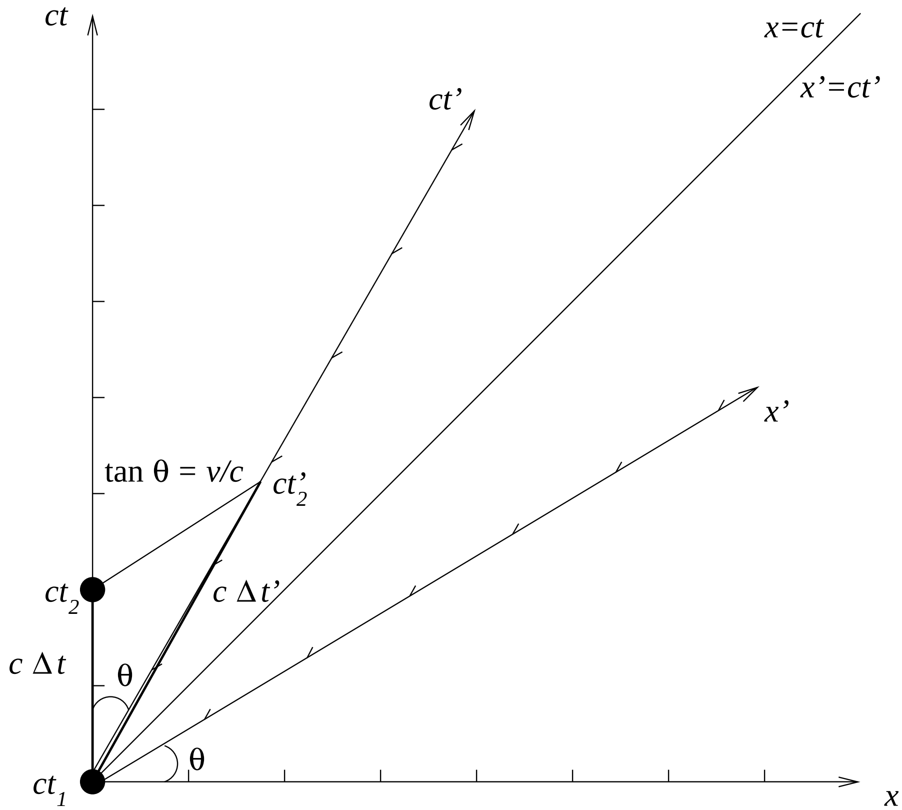


Figure 2: For the two events $x_1 = x_2 = 0$ and $t'_1 = t_1 = 0$. Note: $c\Delta t_{\mathrm{p}} = c\Delta t = 2 < c\Delta t' \approx 2.7$.

- ∘ Consider two events at $x'_1 = x'_2$ at times t'_1 and t'_2. Using the inverse of Eq. (5.1)

$$\Delta t = t_2 - t_1 = \gamma(t'_2 - t'_1) = \gamma \Delta t_{\rm p}. \quad (5.6)$$

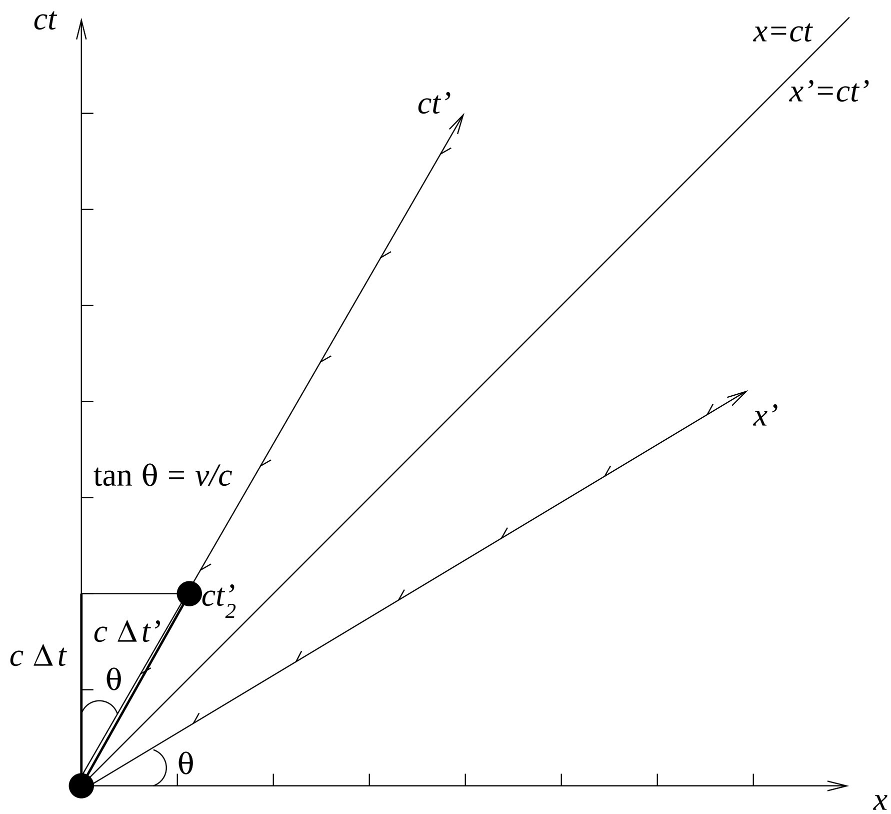


Figure 3: For the two events $x'_1 = x'_2 = 0$ and $t'_1 = t_1 = 0$. Note: $c\Delta t_{\rm p} = c\Delta t' \approx 1.7 < c\Delta t = 2$.

- Length contraction:

  - Consider $L_{\mathrm{p}} = x_2 - x_1$ at same time $t_1 = t_2$, with L' determined by the two events at the same time $t'_1 = t'_2$ at positions x'_1 and x'_2. Then $x_1 = \gamma(x'_1 + vt'_1)$ and $x_2 = \gamma(x'_2 + vt'_2)$. Hence,

$$L_{\mathrm{p}} = x_2 - x_1 = \gamma(x'_2 - x'_1) = \gamma L'. \quad (5.7)$$

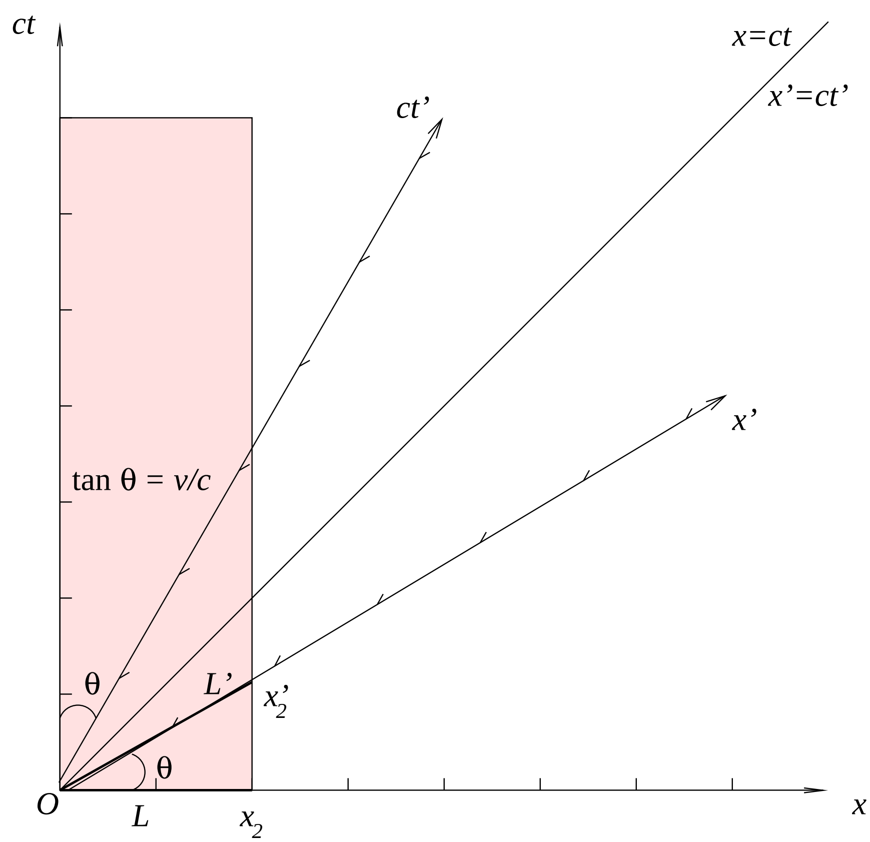


Figure 4: $L_{\mathrm{p}} = L = 2 > L' \approx 1.7$

∘ Consider $L_\mathrm{p} = x'_2 - x'_1$ at same time $t'_1 = t'_2$, with L determined by the two events at the same time $t_1 = t_2$ at positions x_1 and x_2. Using Eq. (5.2)

$$L_\mathrm{p} = x'_2 - x'_1 = \gamma(x_2 - x_1) = \gamma L. \quad (5.8)$$

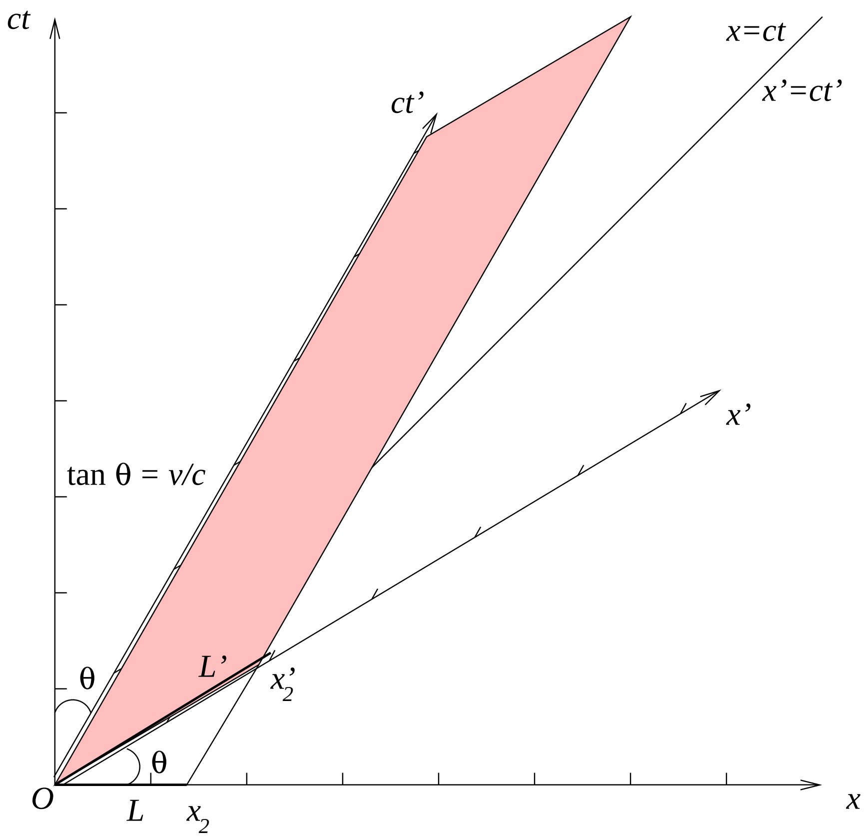


Figure 5: $L_\mathrm{p} = L' \approx 1.9 > L \approx 1.3$

- Same time dilation and length contraction relations are obtained irrespective of frame of reference.

- Rocket with $v = 0.7c$ ($\gamma = 1.4$) travels towards Proxima Centauri, $\Delta x = 4.2$ (OA) light-years away from Earth. Interactive Minkowski Diagram:
  - Earth: Proxima Centauri (C) is reached in $c\Delta t = c\Delta x/v = 6$ light-years, or $t = 6$ years.
  - Rocket: C reached in $\Delta t' = 4.3$ years, traveling $\Delta x' = 3$ (OB) light-years. Note, $v' = \Delta x'/\Delta t' = -3c/4.3 = -0.7c = -v$.

Interactive Minkowski Diagram

$$ct' = \gamma\left(ct - \frac{v}{c}x\right),\ x' = \gamma\left(x - \frac{v}{c}ct\right)$$

$$ct = \gamma\left(ct' + \frac{v}{c}x'\right),\ x = \gamma\left(x' + \frac{v}{c}ct'\right)$$

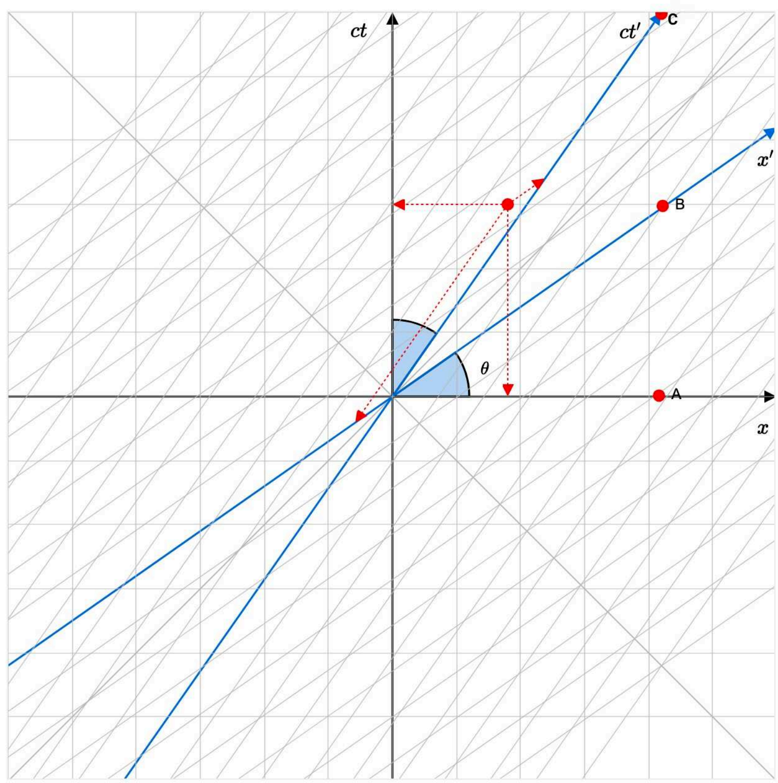
$$\frac{v}{c} = \tan\theta, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

v/c	γ	θ (deg)
0.70	1.40	35

Origins O, O' coincide at $ct = ct' = 0$.
Axes have units of length (e.g. light-years).

- ct axis yields $\Delta t' = \gamma\Delta t = \gamma\Delta t_{\mathrm{p}}$,
- ct' axis yields $\Delta t = \gamma\Delta t' = \gamma\Delta t_{\mathrm{p}}$,
- x axis yields $\Delta x' = \gamma\Delta x = L_{\mathrm{p}}$,
- x' axis yields $\Delta x = \gamma\Delta x' = L_{\mathrm{p}}$.

label	(x, ct)	(x', ct')
A	(4.2, 0.0)	(5.8, -4.1)
B	(4.2, 3.0)	(3.0, 0.0)
C	(4.2, 6.0)	(0.0, 4.3)



5.2 Spacetime and causality

- Define the spacetime interval $(\Delta s)^2$ (for one spatial dimension) via

$$\begin{aligned}(\Delta s)^2 &\equiv (c\Delta t)^2 - (\Delta x)^2 \\ &= (c\Delta t')^2 - (\Delta x')^2 \\ &\equiv (\Delta s')^2,\end{aligned} \tag{5.9}$$

i.e. remains invariant under Lorentz transformations. Verify (5.9) with $\Delta x' \equiv x'_2 - x'_1$ and $\Delta t' \equiv t'_2 - t'_1$ satisfying (5.10) and (5.11), respectively.

  - $(\Delta s)^2 = 0$: null interval ($\Delta x = c\Delta t$), the two events can be connected by a ray of light.
  - $(\Delta s)^2 < 0$: space-like interval, the events cannot be causally connected.
  - $(\Delta s)^2 > 0$: time-like interval, implies the two events may be causally connected, i.e. those events that are separated by a velocity $v < c$. *Worldlines* represent evolution in time of objects, all have $(\Delta s)^2 > 0$.

- Tachyons, mythical faster than light ($\gamma \in \mathbb{C}$) particles, have $(\Delta s)^2 < 0$ and so would lead to causality violation. Nevertheless, they have been "discovered" several times!

Without loss of generality, suppose the tachyons have infinite speed: can be anywhere in space at the one time. Can set up an experiment where tachyons go back in time!

- Suppose two Observers, O and O' are both armed with a tachyon gun and receiver.
- They agree that, as they move apart at high speed, O will fire a tachyon at O'. Upon receipt, O' will fire a tachyon back at O.
- The outcome is that O receives a tachyon before firing one!

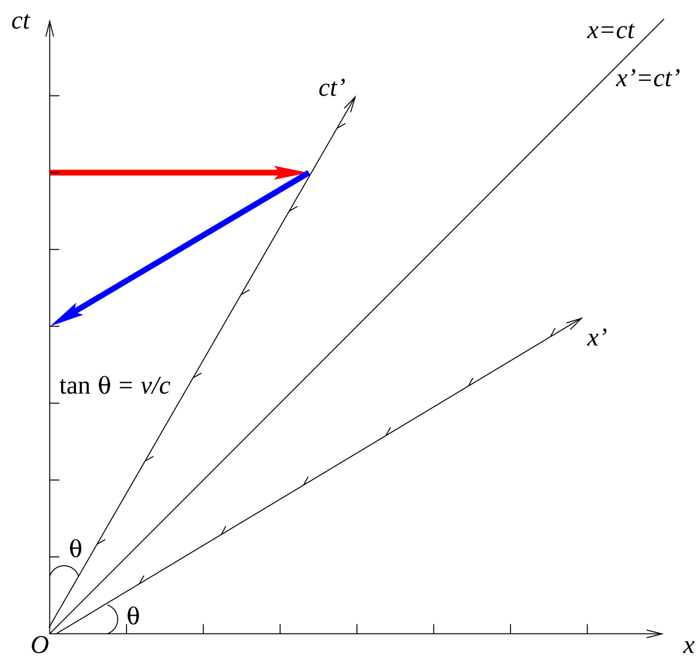


Figure 6: Arrows depict tachyons with infinite speed.

5.3 Lorentz velocity addition

- Using (5.2) and (5.1) with $u_{x'} \equiv dx'/dt'$ and $u_x \equiv dx/dt$, obtain Lorentz velocity addition formulas:

$$\begin{aligned} dx' &= \gamma(dx - vdt) \qquad (5.10) \\ dt' &= \gamma\left(dt - \frac{vdx}{c^2}\right), \qquad (5.11) \\ \frac{dx'}{dt'} &= \frac{dx - vdt}{dt - \frac{vdx}{c^2}} \\ u_{x'} &= \frac{u_x - v}{1 - \frac{vu_x}{c^2}}, \text{ or } u_x = \frac{u_{x'} + v}{1 + \frac{vu_{x'}}{c^2}}. \qquad (5.12) \end{aligned}$$

  - For $|vu_x|/c^2 \ll 1$, $u_{x'} \approx u_x - v$, see (3.2).
  - For $u_x = c$ get $u_{x'} = c$, even for $v = -c$!
  - Similarly, from $dy' = dy$ and $dz' = dz$, have

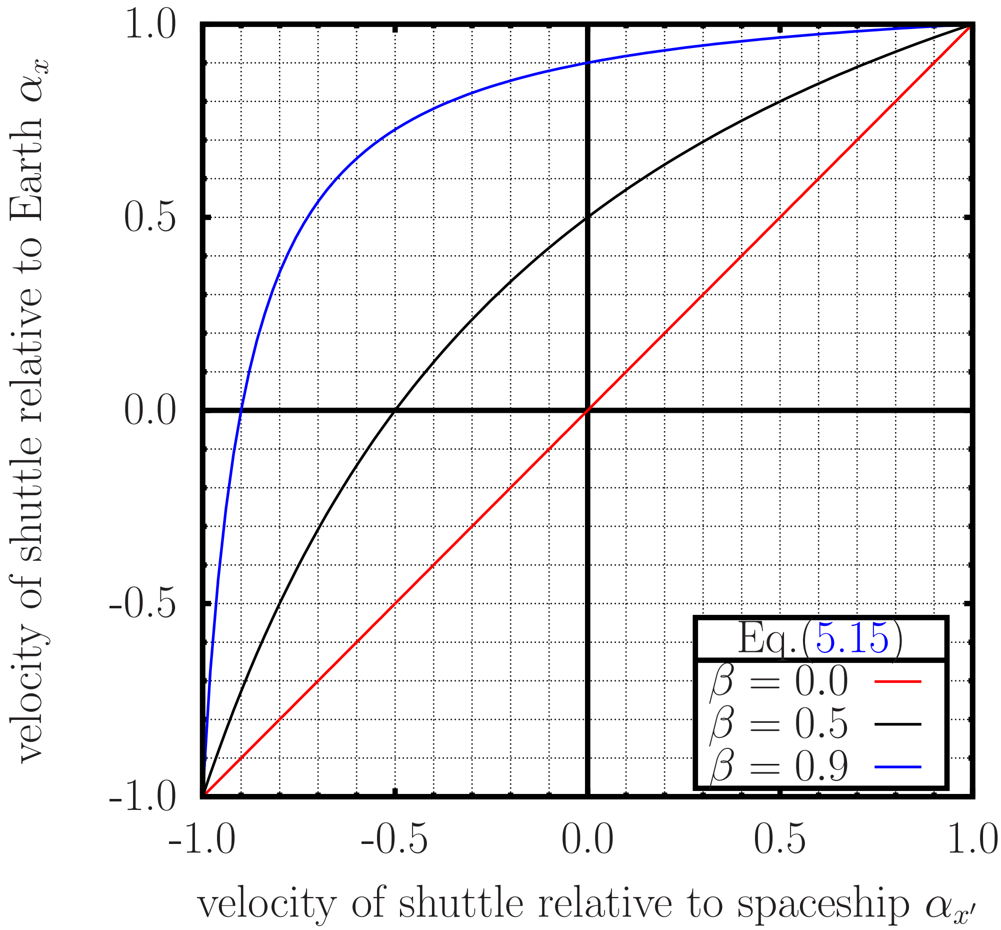
$$\begin{aligned} u_{y'} &= \frac{dy}{\gamma\left(dt - \frac{vdx}{c^2}\right)} \\ &= \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}, \qquad (5.13) \\ u_{z'} &= \frac{u_z}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}. \qquad (5.14) \end{aligned}$$

  - Upon $S \leftrightarrow S'$ with $v \to -v$ obtain formulas for $\boldsymbol{u}$ directly, or via algebra, see (5.12) for u_x.

- For $\alpha_x = u_x/c$, $\alpha_{x'} = u_{x'}/c$ and $\beta = v/c$, Eqs. (5.12) yield Lorentz dimensionless colinear velocity addition formulas

$$\alpha_{x'} = \frac{\alpha_x - \beta}{1 - \alpha_x \beta} \text{ or } \alpha_x = \frac{\alpha_{x'} + \beta}{1 + \alpha_{x'} \beta}. \qquad (5.15)$$

- For spaceship moving with velocity v relative to the Earth, and shuttle moving colinearly with velocity $u_{x'}$ relative to the spaceship, the velocity of the shuttle relative to the Earth u_x never exceeds c ($|\alpha_x| \leq 1$):



6 Relativistic Laws of Physics

We saw how the Galilean velocity transformations $u_{x'} = u_x - v$ ensured that the Newtonian momentum conservation law was covariant under the Galilean transformations, see Eq.(3.3). The Lorentz velocity transformation (5.12) destroys this.

6.1 Momentum, force and energy

- For proper time $\tau = t/\gamma$ define relativistic momentum vector $\boldsymbol{p}$ for velocity $d\boldsymbol{x}/dt \equiv \boldsymbol{u} = u\hat{\boldsymbol{u}}$, where $\hat{\boldsymbol{u}}$ is a unit vector, by

$$\boldsymbol{p} \equiv m\frac{d\boldsymbol{x}}{d\tau} = m\frac{d\boldsymbol{x}}{dt}\frac{dt}{d\tau} = \frac{m\boldsymbol{u}}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma(u)m\boldsymbol{u}. \tag{6.1}$$

- Relativistic force vector $\boldsymbol{F}$ is defined by

$$\begin{aligned} \boldsymbol{F} \equiv \frac{d\boldsymbol{p}}{dt} &= m\hat{\boldsymbol{u}}\frac{du}{dt}\frac{d}{du}\left(\gamma(u)u\right) \\ &= m\boldsymbol{a}\frac{d}{du}\left(\gamma(u)u\right) \\ &= m\boldsymbol{a}\gamma^3(u). \end{aligned} \tag{6.2}$$

For a constant F, as u increases towards c the acceleration decreases!

- Using (6.2) with $\boldsymbol{F}$ in direction $\boldsymbol{x}$, the relativistic energy is defined from the work-energy relation

$$W \equiv \int_{x_1}^{x_2} F dx = m \int_{x_1}^{x_2} \frac{\frac{du}{dt} dx}{\left(1 - \frac{u^2}{c^2}\right)^{\frac{3}{2}}}, \text{ as } \frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt},$$

$$= m \int_{u_1}^{u_2} \frac{u du}{\left(1 - \frac{u^2}{c^2}\right)^{\frac{3}{2}}}, \text{ where } u du = \frac{dx}{dt}\frac{du}{dx} dx,$$

$$= \left.\frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}}\right]_{u_1}^{u_2} = \frac{mc^2}{\sqrt{1 - \frac{u_2^2}{c^2}}} - \frac{mc^2}{\sqrt{1 - \frac{u_1^2}{c^2}}}. \quad (6.3)$$

  - Without loss of generality we take $u_1 = 0$ then $K = W$ is the relativistic kinetic energy

$$K = \gamma mc^2 - mc^2. \quad (6.4)$$

  - Using Eq.(4.2) for $u \ll c$ have $K \approx \frac{1}{2} mu^2$.
  - Define the total energy $E = K + mc^2$, and so

$$E = \gamma mc^2. \quad (6.5)$$

  - mc^2 is the rest energy.

- Historically, m was defined as the rest mass m_0, and $m = \gamma m_0$ as the relativistic mass leading to

$E = mc^2$ as the general case, and not just the rest case. This is no longer in fashion. Eq. (6.5) is the general case, where m is called the invariant mass.

6.2 Momentum-energy conservation

- Goal: to formulate relativistic energy and momentum conservation laws within an inertial frame of reference.

- From relations $E = \gamma mc^2$ and $p = \gamma mu$ can easily show that (please do)

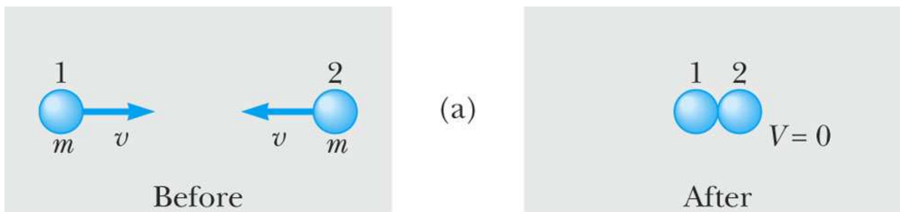
$$E^2 = p^2c^2 + m^2c^4. \tag{6.6}$$

  - For $p = 0$ obtain the rest energy $E = mc^2$.
  - As $u \to c$ have $E \to pc$.
  - For photons $u = c,\ m = 0$, so $E = pc$. Define photon momentum p, in direction of propagation, from Planck's photon energy $E = hf$

$$p = hf/c. \tag{6.7}$$

- Note, since $E^2 - p^2c^2 = m^2c^4$, this must be invariant under Lorentz transformations, but not E and p separately since they depend on u.

- The law of Mass-Energy conservation states that E before and after an interaction must be the same (within an inertial frame of reference).
  - Fusion: Suppose two particles of mass m are moving toward each other with velocities v and $-v$, relative to the centre of mass frame, fuse together after the collision:



  - Classically, the initial total kinetic energy is $K = 2mv^2/2$, and the final $K = 0$.
  - Relativistically, have $E = 2\gamma mc^2 = Mc^2$, i.e.

$$M = 2\gamma m \geq 2m. \tag{6.8}$$

  - Define $\Delta M \equiv M - 2m = M(\gamma - 1)/\gamma = 2m(\gamma - 1) = 2K/c^2$, using Eq. 6.4.
  - Fission: A mass M breaks into two equal parts of $m = 0.4M$ with velocities v and $-v$ then

$$Mc^2 = 2\gamma 0.4Mc^2, \gamma = \frac{1}{0.8}, v = 0.6c.$$

- Since E is conserved within a frame, and $p^2c^2 = E^2 - m^2c^4$, have p automatically conserved as well!